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Info-Gap Approach to Manage GenCo's Trading Portfolio with Uncertain Market Returns

Parul Mathuria, Rohit Bhakar, *Member, IEEE*

Abstract—An independent Generation Company (GenCo) secures its future trading position by managing its portfolio among multiple trading options. Future returns of these trading options are not known during decision making and are traditionally estimated using probabilistic or fuzzy methods. Quantifying such uncertainty of market returns by conventional methods does not reflect the information gap existing between estimated and actual market returns. Based on quantification of this information gap, the paper proposes GenCo's portfolio optimization using a non-probabilistic Information Gap Decision Theory (IGDT). This framework comprehensively models GenCo's behavior in deciding its trading strategy. Considering GenCo's risk-averse behavior, the framework provides decisions that are robust towards losses, while considering its risk-seeking behavior the framework offers opportunity to capture windfall gains. The proposed approach has been validated through practical case study of PJM market.

Index Terms—GenCo, information gap decision theory, portfolio optimization, uncertainty.

I. NOMENCLATURE

A. Indices

i, j	Index of trading contract
k	Index of trading interval

B. Parameters

a	No-load heat-rate coefficient in MW
b	Linear heat-rate coefficient in MW/MBtu
c	Quadratic heat-rate coefficient in MW/MBtu ²
$LMP_{i,k}$	LMP of trading area i in k^{th} trading interval
M	Considered time horizon or planning period
n	Number of locations
$p_{i,k}^{Min}$	Minimum trading limit for contract i in k^{th} trading interval
$p_{i,k}^{Max}$	Maximum trading limit for contract i in k^{th} trading interval
r	Set of uncertain returns from contracts ($i = 1 \sim n$)
$\tilde{\cdot}$	Estimate of returns from contract
r_C	Critical return
r_W	Windfall return
t	Time for each trading interval in hours

γ	Congestion charge factor, varying from 0 to 1
$\lambda_{i,k}$	Price of trading contract i in k^{th} trading interval (\$/MWh)
$\hat{\lambda}_{i,k}$	Forecast/Estimate of i^{th} contract price at k^{th} trading interval
$\lambda_{i,k}^B$	Bilaterally agreed price for contract i during k^{th} trading interval (\$/MWh)
λ_k^F	Fuel price in k^{th} trading interval (\$/MBtu)
μ	Lagrange coefficient

C. Decision Variables

$p_{i,k}$	Traded power for contract i in k^{th} trading interval
$u_{i,k}$	Binary variable representing selection state of contract i in k^{th} trading interval
w	Set of weights reflecting allocation in each trade
α	Uncertainty parameter or Horizon of uncertainty

D. Functions

$C(p_{i,k}, \lambda_k^F)$	Generation cost for trading quantity $p_{i,k}$
$R(q, u)$	System model in IGDT method
$R_p(w, r)$	Portfolio return for allocation w and return r
$RV(p_{i,k}, \lambda_{i,k})$	Revenue from i^{th} contract
$U(\alpha, u)$	Uncertainty function in IGDT method
$\alpha(q, r_C)$	Robustness function in IGDT method
$\beta(q, r_w)$	Opportuneness function in IGDT method

II. INTRODUCTION

DEREGULATION of power sector has introduced a variety of markets and trading alternatives, offering multiple choices for independent power companies to trade electricity. In electricity markets, these companies make advance trading decisions. During real time condition, prices are affected by several unpredictable factors, causing them to vary widely and creating high market uncertainty. This affects trading contracts in electricity market and makes them uncertain, leading to unique price-volatility characteristic for each contract. GenCos as a supplier of electricity, appropriately diversify their trading in multiple trading options to reduce their risk exposure and maximize profit [1]. This strategic decision making to allocate energy among multiple contracts, considering risk-return trade-off in a prospective market and risk preference of the company is known as portfolio optimization.

Parul Mathuria is associated with Electrical Engineering Department, Malaviya National Institute of Technology Jaipur, India. (e-mail: parulvj14@gmail.com)

Rohit Bhakar is Prize Fellow, Department of Electronic and Electrical Engineering, University of Bath, Bath, UK. (e-mail: r.bhakar@bath.ac.uk).

In electricity markets, portfolio optimization is an important mechanism of risk hedging, considering the correlation between different trading options [1]-[5]. GenCos use it to secure themselves from multiple uncertainties of wholesale markets involving physical and financial trading [3]-[5]. These may include pool price uncertainty, fuel price uncertainty [6], transmission congestion charges uncertainty [7], *etc.* For portfolio selection, conventional approaches like Markowitz mean-variance theory [6]-[10], stochastic programming using Value at Risk (VaR) [5] and Conditional Value at Risk (CVaR) [11], have been used.

Existing approaches make decisions relying on estimation of future conditions and use probability distribution or fuzzy membership functions [1]-[11]. Estimations are based on historical data. However, past returns are an unreliable guide to estimate future returns, and quantifying uncertainty by probability distribution or otherwise requires information about future behavior of market returns. Portfolio involves multiple uncertain trading options. An optimal portfolio selection requires precise forecast of several input parameters, such as individual return, variance and correlation between different uncertain options. Such precise forecasts are not available during decision making and erroneous estimation gets reflected as inappropriate diversification. This leads to severe losses due to the large quantum of involved power. Further, the existing approaches make decisions based simply on risk return trade-off for portfolio selection, considering risk-averse nature of decision makers. However, practical market scenario necessitates broader criteria to be considered for improving portfolio performance that meets GenCo's aspirations.

In sharp contrast to existing portfolio selection approaches, IGDT provides reliable decisions based on quantification of information gap between estimated and actual value, without depending upon estimates. It can help to construct portfolios that are robust against losses as well as opportunistic to capture windfall gains [12]. Huge volatility of electricity markets and necessity of advance decision making makes IGDT an attractive option to understand and solve a wide variety of market issues, *viz.* electricity purchase bidding, robust scheduling of large consumers, and GenCo's self-scheduling and optimal bidding [13]-[16].

This paper proposes a quantitative methodology based on IGDT, to deal with severe uncertainty during portfolio selection, involving assets with uncertain returns. These assets are options available for GenCo to trade electricity in spot and contract market, considering price uncertainty of pool and congestion charges [7]. The proposed formulation deals with uncertainty of each trading alternative and their co-movements. It considers risk averse as well as risk seeking nature of decision maker. The results based on a case study from PJM market highlight trade-offs existing between reward and robustness, opportunity and windfall gain, and robustness and opportuneness, for portfolio selection.

III. IGDT FRAMEWORK

IGDT is a non-probabilistic decision making theory, which seeks to optimize robustness to failure and opportuneness to capture windfall gains, under conditions of deficient information about parameter of interest. In portfolio selection, information about nominal estimates of uncertain asset returns and their relative degree of variation is evaluated from past data. However, actual values of these parameters may deviate from estimated ones with unknown quantum of deviation. IGDT models size of gap between estimated and actual value, as uncertainty parameter α , which is allowed to be unbounded [17].

IGDT evaluates decisions at many points, as uncertainty varies from estimation in an unbounded manner. These points are different values of uncertainty parameter α evaluated from different performance requirements. It helps the decision maker to compare different trading decisions which satisfy system performance criteria as per its requirements or aspirations. An IGDT decision making problem is specified by three component models:

A. System Model

For a set of decision variables q and uncertain parameter u , the system model $R(q, u)$ expresses the input/output structure of the system based on which the decisions are evaluated [17]. Here, u is input parameter of interest which deviates from its nominal estimate in an unknown transient manner. System model is the objective function, *i.e.* GenCo's portfolio return which is obtained for certain energy allocation in available trading alternatives.

B. Uncertainty Model

Uncertainty sets have a common structure which depends upon available information about parameter of interest [17]. Uncertainty U of an uncertain input parameter u can be defined as an unbounded family of nested sets, nested by the uncertainty parameter α around estimate u . Ellipsoid bound model of info-gap uncertainty can be used to model variability in estimates and degree of co-variability of different securities. This is mathematically represented as [17]

$$U(\alpha, u) = \{u = u + \Delta u : \Delta u^T C^{-1} \Delta u \leq \alpha^2\}, \quad \alpha \geq 0 \quad (1)$$

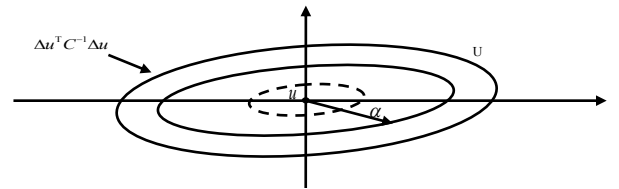


Fig. 1 Ellipsoid bound info-gap model of uncertainty representing unbounded uncertainty as a family of nested sets

Here, C represents covariance matrix between different assets. The quadratic term defines an ellipsoid, centered at estimate u . Size of each ellipsoid is characterized by free uncertainty parameter α . The ellipsoid represents envelope for uncertainty, *i.e.* region defined by $U(\alpha, u)$ within which all u would lie, for a particular α .

C. Performance Requirements

The performance requirements express values of the outcomes that the decision maker requires or aspires to achieve, while selecting a decision [12]. The decision maker may consider uncertainty to be pernicious or propitious, depending upon its risk averse or risk seeking nature. The two conflicting concepts: satisfying or meeting critical requirements and aspiring windfall goal for better-than-expected outcomes, are evaluated based on robustness and opportuneness functions respectively [12]. The robustness function guarantees a certain return, while opportuneness function identifies the possibility of securing windfall benefit. Both the functions optimize uncertainty parameter α such as

$$\alpha(q, r_c) = \max\{\alpha : \min R(q, u) > r_c\} \quad (2)$$

$$\beta(q, r_w) = \min\{\alpha : \max R(q, u) > r_w\} \quad (3)$$

The robustness function $\alpha(q, u)$ expresses the maximum uncertainty (α) as (2), at which minimum requirements are always satisfied *i.e.* minimum return should always be more than r_c . This addresses conservative nature of decision maker and expresses the level of protection for the selected decision, in case the market falls.

Opportunity function models the nature of optimistic decision maker who wish to benefit from favorable market changes. Opportuneness function $\beta(q, u)$ represents the minimum level of uncertainty which has to be tolerated to achieve windfall return as large as r_w . Both functions evaluate uncertainty parameter α in order to obtain required or aspired outcomes.

IV. PROBLEM DESCRIPTION AND FORMULATION

This paper presents an IGDT-based formulation for GenCo's power portfolio optimization in medium term time frame. The work considers physical trading approaches, involving pool and bilateral contract markets under Locational Marginal Pricing (LMP) scheme. The GenCo makes bilateral contracts with consumers situated in same or different location. Bilateral contracts of different location may be affected by congestion [7]. Congestion creates LMP separation between locations. The difference between LMPs of two locations (where generator and load are connected), are applicable congestion charges for underlying contract. These charges are to be paid by supplier or consumer proportionately depending upon market rule that defines this proportion. In case of spot market trading, GenCo receives LMP of its own location.

LMPs are based up on real time network conditions and can hardly be predicted at the time of planning. Thus, contracts that depend on LMPs are uncertain and returns from such contracts can only be estimated during planning. The returns of various uncertain contracts available to GenCo are correlated with each other up to a certain degree. This can be considered as an appropriate case for portfolio optimization in electricity market, as consideration of congestion and pool

uncertainty provides the widest variety of trading options, each with its unique return-volatility characteristics and different correlations. Bilateral contract specifications (contract price, quantity, time *etc.*) are assumed known to decision maker. Returns from bilateral contracts within home location are not affected by network constraints, and are thus considered deterministic. This portfolio selection problem of allocating energy among multiple assets has been formulated for a price taker GenCo in an IGDT framework, for dealing severe uncertainty of returns from different trading options. The presented formulation is restricted to consideration that markets are efficient, competitive and sufficiently liquid. This work considers day-ahead market to represent pool, however similar modelling can be extended to integrate other markets (intraday and balancing) and trading contracts as well.

For simplicity in calculations, it is assumed that a GenCo would make only one bilateral contract with consumer of a certain location and there exists a single spot market. For n considered locations, GenCo can have a total of $n+1$ contracts; one bilateral contract with consumer of home location and $n-1$ bilateral contracts with consumer of non-home locations and one spot market contract.

A. Contract Price Modelling

A GenCo is assumed to be placed at Location 1. For the local bilateral contract (indexed as $i = 0$), contract price would be equal to bilaterally agreed price $\lambda_{1,k}^B$ with consumer of home location, as transmission charges are not applicable.

$$\lambda_{0,k} = \lambda_{1,k}^B \quad \forall k \quad (4)$$

In spot market trading (indexed as $i = 1$), GenCo would receive LMP of its own location *i.e.* Location 1, where generators inject power into the system. Their contract price would be

$$\lambda_{1,k} = LMP_{1,k} \quad \forall k \quad (5)$$

Contracts indexed as $i = 2 \sim n$ represent bilateral contracts with non-home locations. For this, applicable per unit congestion charges for transmitting energy from location 1 to location 2, at time k , would be:

$$\text{Congestion charges} = LMP_{2,k} - LMP_{1,k} \quad \forall k \quad (6)$$

Contract-holders pay congestion charges on such contracts proportionately, based on γ ($0 \leq \gamma \leq 1$), as per the market rule. So, the effective contract prices for GenCo are

$$\lambda_{i,k} = \lambda_{i,k}^B - \gamma(LMP_{i,k} - LMP_{1,k}) \quad \forall k \quad (7)$$

for $i = 2 \sim n$

Thus, except for local bilateral contract (indexed as $i = 0$), all contracts depend upon severely uncertain LMPs, which make contract prices uncertain. Based on prices of different trading options, returns from each can be calculated to evaluate overall portfolio return.

B. Portfolio Return

Overall portfolio return R_p is the weighted sum of individual returns from each trade. Return from local bilateral contract is indexed as zero. Future net return of the portfolio

with $n+1$ assets, with returns r_i and relevant weight w_i is:

$$R_p = \sum_{i=0}^n w_i r_i \quad (8)$$

$$s.t. \quad \sum_{i=0}^n w_i = 1 \quad (9)$$

$$\text{and } w_i \geq 0 \quad (10)$$

Return from any asset can be considered as

$$\text{Return} = \frac{\text{Revenue} - \text{Cost}}{\text{Cost}} \quad (11)$$

Revenue generated from trading $p_{i,k}$ power in each option i at contract price $\lambda_{i,k}$ can be calculated for each trading interval as

$$RV(p_{i,k}, \lambda_{i,k}) = t p_{i,k} \lambda_{i,k} \quad (12)$$

Generation cost is composed of two components fixed and variable cost. It can be represented as quadratic cost curve $C(p_k, \lambda_k^F) = (a + bp_k + cp_k^2)t\lambda_k^F$, where p_k is the generation outcome (MW). Generated power at each trading interval is then allocated among $n+1$ trading options. Thus, generation cost for p_k is divided for power traded in each option. Share of cost for power traded¹ in i^{th} contract $p_{i,k}$, at k^{th} trading interval can be calculated as:

$$C(p_{i,k}, \lambda_k^F) = t b_k \lambda_k^F p_{i,k} \quad (13)$$

where $b_k = \frac{a + bp_k + cp_k^2}{p_k}$, and fuel prices are assumed deterministic for the considered planning period.

Using (12) and (13), return from each trading contract i ($i=1 \sim n$) for planning period M^2 , can be evaluated by averaging out the returns of each trading interval k as

$$r_i = \frac{1}{M} \sum_{k=1}^M \frac{t(\lambda_{i,k} - b_k \lambda_k^F) p_{i,k}}{t b_k \lambda_k^F p_{i,k}} \Rightarrow r_i = \frac{1}{M} \sum_{k=1}^M \frac{\lambda_{i,k}}{K_k} - 1 \quad (14)$$

$$\text{where } K_k = b_k \lambda_k^F \quad (15)$$

C. IGDT Formulation

The discussed portfolio optimization problem is formulated based on IGDT framework of Section III. The future returns from different trading options r_i and corresponding proportion of energy allocation w_i evaluate portfolio return R_p . Uncertain returns from different trading options, except r_0 , are considered as uncertain parameters of the problem. The corresponding weights are decision variables. For the considered uncertainty model, trading strategies are derived for robustness and opportunity functions, using system model and relevant constraints.

1) System Model

For the considered planning period, net future portfolio return of known (return from local bilateral contract (r_0)) and

unknown (returns from contracts $i=1 \sim n$) assets is considered as

$$R_p(w, r) = w_0 r_0 + \sum_{i=1}^n w_i r_i \quad (16)$$

If $w = [w_1 \ w_2 \ \dots \ w_n]$ and $r = [r_1 \ r_2 \ \dots \ r_n]$, then (16) can be re-written as

$$R_p(w, r) = w_0 r_0 + w^T r \quad (17)$$

As r is uncertain, it can deviate Δr from its expected value \tilde{r} as

$$r = \tilde{r} + \Delta r \quad (18)$$

where $\tilde{r} = [\tilde{r}_1 \ \tilde{r}_2 \ \dots \ \tilde{r}_n]$ and $\Delta r = [\Delta r_1 \ \Delta r_2 \ \dots \ \Delta r_n]$

Variation in returns is due to variation in contract prices $\Delta \lambda_{i,k}$ from their expected value $\lambda_{i,k}$. Corresponding relation can be represented as

$$r_i = \frac{1}{K_k} \sum_{k=1}^M (\lambda_{i,k} + \Delta \lambda_{i,k}) - 1 \quad (19)$$

The system model or portfolio return can be represented as

$$R_p(w, r) = w_0 r_0 + w^T (\tilde{r} + \Delta r) \quad (20)$$

2) Uncertainty Model

A typical historical covariance matrix for different asset returns ($i=1 \sim n$) contains information indicating the relative degree of variability in prices of different trading options and their correlated or anti-correlated variations. An ellipsoid bound model of IGDT uses this information to formulate uncertainty of returns, by forming uncertainty shape matrix C . The model can be used without any correlation between securities, with only diagonal matrix elements. Using the available information, uncertainty can be modelled as

$$U(\alpha, \tilde{r}, C) = \{ \tilde{r} + \Delta r \mid \Delta r^T C \Delta r \leq \alpha^2 \} \quad \alpha \geq 0 \quad (21)$$

Here, size of C is $n \times n$, where n represents the number of uncertain trades. C is real, symmetric and positive definite matrix. The matrix elements can be calculated using (14) and (15) as:

$$C_{i,j} = \text{cov}(r_i, r_j) \quad (22)$$

The covariance matrices are evaluated between uncertain returns. Covariances between these returns are evaluated for each trading interval and averaged out to obtain elements of C for planning period M

$$C_{i,j} = \frac{1}{M} \sum_{k=1}^M \frac{\text{cov}(\lambda_{i,k}, \lambda_{j,k})}{K_k^2} \quad (23)$$

for $i = \{1, 2, \dots, n\}$ and $j = \{1, 2, \dots, n\}$

3) Robustness Function

A risk-averse GenCo wishes to immune itself from losses. Robustness function evaluates the level of protection for certain fall in market returns ensuring that minimum return would not be less than a critical return r_c . It evaluates the level of uncertainty that any decision can sustain without sacrificing certain performance requirements. Robustness of the portfolio selection strategy w , for critical return r_c , is the largest value of uncertainty parameter α , such that any return in

¹ Traded quantity for each transaction is the product of corresponding allocation w_i and total scheduled power p .

² Planning period could be one day, one month or one year.

region $U(\alpha, \tilde{r}_w)$ would give $R_p(w, r)$, which is not less than r_c . For performance requirement (2) to be satisfied, for a certain uncertainty α , when all $r \in U(\alpha, \tilde{r}_w)$, the minimum value of portfolio return would be

$$\min_{\Delta r} R_p(w, r) = w_0 r_0 + w^T \tilde{r}_w \quad (24)$$

$$s.t. \Delta r^T C^{-1} \Delta r \leq \alpha^2 \quad (25)$$

These equations can be rewritten as

$$\min_{\Delta r} R_p(w, r) = w_0 r_0 + w^T \tilde{r}_w \quad \text{subject to } \Delta r^T C^{-1} \Delta r \leq \alpha^2 \quad (26)$$

Above optimization problem can be solved using Lagrange relaxation method, which gives (derivation in Appendix A)

$$\Delta r = \pm \alpha \frac{C w}{\sqrt{w^T C w}} \quad (27)$$

So, for minimum $R_p(w, \alpha)$, negative value of Δr is used (as $\alpha > 0$), which results in:

$$\min R_p(w, \alpha) = w_0 r_0 + w^T \tilde{r}_w - \alpha \sqrt{w^T C w} \quad (28)$$

Minimum portfolio return should be at least equal to r_c .

$$w_0 r_0 + w^T \tilde{r}_w - \alpha \sqrt{w^T C w} = r_c \quad (29)$$

$$\Rightarrow \alpha(r_c) = \frac{w_0 r_0 + w^T \tilde{r}_w - r_c}{\sqrt{w^T C w}} \quad (30)$$

Robustness function represents the largest value of α for a targeted return r_c .

$$\alpha(w, r_c) = \max_w \alpha(r_c) = \max_w \frac{w_0 r_0 + w^T \tilde{r}_w - r_c}{\sqrt{w^T C w}} \quad (31)$$

As α is the size of uncertainty ellipsoid, and represents gap from nominal estimate, it cannot be negative. So for $r_c > (w_0 r_0 + w^T \tilde{r}_w)$, it is zero.

For portfolio optimization, optimal energy allocation strategy depends on returns and uncertainty of assets, as well as their relative propensity of variation. Robustness decreases with increase in targeted critical returns. Optimal weights for maximizing robustness $\alpha(w, r_c)$ for GenCo's energy allocation are so selected that portfolio return is maximized, while $w^T C w$ is minimized, subject to other constraints.

4) Opportunity Function

To benefit from the opportunity of high market prices, a GenCo has to bear certain uncertainty. Opportunity $\beta(w, r_w)$ is the least level of uncertainty which must be tolerated in order to enable the possibility of reward as large as r_w . The maximum possible return up to uncertainty α , when all $r \in U(\alpha, \tilde{r}_w)$, subject to (21) for $\alpha > 0$, can be calculated using Lagrange method, considering positive value of Δr from (27).

$$\max_{\Delta r} R_p(w, r) = w_0 r_0 + w^T \tilde{r}_w + \alpha \sqrt{w^T C w} \quad (32)$$

Maximum value of $R_p(w, \alpha)$ should be as large as windfall return r_w , thus

$$w_0 r_0 + w^T \tilde{r}_w + \alpha \sqrt{w^T C w} = r_w \quad (33)$$

$$\Rightarrow \alpha(r_w) = \frac{r_w - (w_0 r_0 + w^T \tilde{r}_w)}{\sqrt{w^T C w}} \quad (34)$$

Based on (3), this represents least possible uncertainty. So, using (34), opportunity function can be evaluated as

$$\beta(w, r_w) = \min_w \alpha(r_w) = \min_w \frac{r_w - (w_0 r_0 + w^T \tilde{r}_w)}{\sqrt{w^T C w}} \quad (35)$$

Opportunity function $\beta(w, r_w)$ increases with windfall returns. For a certain value of r_w , opportunity $\beta(w, r_c)$ is minimized by maximizing $w^T C w$ and portfolio return.

Finally, (31) and (35) are two optimization problems, which are solved for multiple values of r_c and r_w to decide trading strategy w as per GenCo's nature. $\alpha(w, r_c)$ is maximized when $w^T C w$ is to be minimized while $\beta(w, r_c)$ is minimized for $w^T C w$ to be maximized. Thus, the portfolio optimization strategies offered by two optimization problems are divergent. An optimistic decision maker selects an opportunistic strategy, generally accompanied with higher uncertainty contracts, though recognizing that it would be less robust. Minimization of $w^T C w$ is possible with single plane constraint (5) but its maximization requires additional constraints. In this problem, the limiting constraint on trading contracts is

$$p_{i,k}^{Min} u_{i,k} \leq p_{i,k} \leq p_{i,k}^{Max} u_{i,k} \quad \text{for } \forall i, \forall k \text{ and } (i \neq 1) \quad (36)$$

$$u_{i,k} \in \{0, 1\} \quad \text{for } \forall i, \forall k \text{ and } (i \neq 1) \quad (37)$$

where variable $u_{i,k}$ decides the selection of contract. The two optimization problem (31) and (35), each under the constraints (5), (6), (36) and (37), are MINLP problems. These can be optimized individually for given return targets, in order to obtain asset weights w .

V. CASE STUDY AND RESULTS

To analyse the proposed portfolio optimization methodology for GenCos, a case study of PJM electricity market has been considered [18]. A GenCo of 600 MW total capacity with generation specifications $a = 735.583$ MBtu, $b = 8.28$ MBtu/MW and $c = 0.00115$ MBtu/MW², intends to plan its trading strategy for the month July 2013. The considered planning period is one month, with one hour as trading interval, i.e. a total of $31 \times 24 = 744$ trading intervals. Fuel prices are considered stable at 4 \$/MBtu, for all trading intervals. The GenCo trades its scheduled generation (assumed equal to maximum capacity in this case) in day-ahead spot market and in multiple bilateral contracts with six different locations, as per contract specifications shown in Table I. AECO is considered as home location for the trading GenCo and bilateral contract with it is indexed as Contract 0 ($i = 0$).

LMPs of AECO are spot contract price for GenCo and are indexed as Contract 1 ($i=1$).

TABLE I
SPECIFICATIONS OF BILATERAL CONTRACTS

Contract Number	Location Name	Contract Prices (\$/MWh)	Min. (MW)	Max. (MW)
0	AECO	56	50	300
2	COMED	42	50	300
3	AEP	43.5	50	300
4	PENELEC	49.5	50	300
5	APS	51.5	50	300
6	PECO	54	50	300

Hourly day ahead LMPs of month July, from year 2006 to 2012, have been used to calculate the expected/forecasted LMPs. These calculate expected contract prices $\lambda_{i,k}$ (shown in Fig. 2.) using (4)-(7), based on the specifications shown in Table I and $\gamma=1$. Among the total of $n+1$ contracts, for all returns except Contract 0, expected values $\tilde{\cdot}$ are estimated using expected value of contract prices $\lambda_{i,k}$ (Contracts 1-6) and generation specification is based on relation shown in (19).

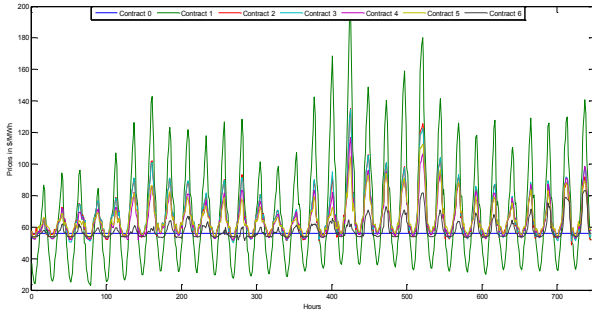


Fig. 2 Estimated contract prices $\lambda_{i,k}$ based on estimated LMPs

Uncertainty shape matrices for each trading interval are calculated from (23), using variance-covariance between n uncertain contract prices, by appropriate function in MATLAB® [19]. For the considered data, there exist 744 matrices of the order 6×6 . All matrices are not shown in the paper due to space limitation.

TABLE II
UNCERTAINTY SHAPE MATRIX BETWEEN RETURNS

Contract Index	1	2	3	4	5	6
1	1.020	0.556	0.550	0.383	0.387	0.163
2	0.556	0.385	0.374	0.255	0.258	0.107
3	0.550	0.374	0.369	0.253	0.252	0.107
4	0.383	0.255	0.253	0.197	0.188	0.097
5	0.387	0.258	0.252	0.188	0.205	0.090
6	0.163	0.107	0.107	0.097	0.090	0.085

Table II shows the uncertainty shape matrix for total planning period, which represents variability and co-variability of returns. The diagonal elements represent individual variability, while off-diagonal elements represent co-variability between returns of uncertain trades. Contract 1 represents high variability, while other contracts have comparatively less variability, with a minimum for Contract 6.

A. Simulations

For the estimated values of input parameters, without considering any uncertainty (*i.e.* $\Delta r = 0$), maximum value of portfolio return $R_P(w, \tilde{\cdot})$ is evaluated by optimizing (20), subject to (5), (6), (36) and (37). For robust decision making, critical return targets r_C are assumed less than $R_P(w, \tilde{\cdot})$, in decreasing small steps. For opportunistic strategies, windfall returns r_W are assumed more than $R_P(w, \tilde{\cdot})$ in increasing small steps. For all r_C less than and for all r_W higher than the maximum obtained portfolio return $R_P(w, \tilde{\cdot})$, optimization is performed for robustness (31) and opportunity (35) respectively, subject to (5), (6), (36) and (37).

For each value of r_C and r_W , a particular trading strategy is obtained in terms of weight w . For the present analysis, each optimization problem has been solved with 6706 real and 5208 discrete variables, using SBB-CONOPT® solver of GAMS in a Core i5, 3.2 GHz processor and 4 GB RAM computer, with an average solution time of 2.7 seconds [20].

With both optimization being MINLP in nature, global optimality is not guaranteed. Approaches available in literature provide global solutions for such problems in different conditions; however this is not the focus of this work.

B. Results

For the above considered case, maximum obtained portfolio return $R_P(w, \tilde{\cdot})$ for zero uncertainty is 0.7785, corresponding to energy allocation w , as shown in Table III. This reflects risk neutral behavior of GenCo, when it takes decision considering estimated value as true value, without considering any uncertainty in future. At $r_C = r_W = 0.7785$ both robustness and opportunity are zero. Values for the two functions increase as the return target varies from 0.7785 in different directions, as shown in Fig. 3. In this situation, the estimations are accurate, so the targeted returns are equal to expected returns.

TABLE III
ENERGY ALLOCATION WITHOUT CONSIDERING UNCERTAINTY

Contract	0	1	2	3	4	5	6
Allocation %	8.5	24.9	15.1	19.6	7.3	23.7	0.9

The two optimization problems (31) and (35) calculate uncertainty/error α from expectation for different values of r_C and r_W . Fig. 3 shows horizon of uncertainty evaluated for robustness and opportunity functions versus targeted returns r_C and r_W . This represents performance quantification under various uncertainty levels, as robustness at any demanded return and as opportuneness for windfall returns, considering two faces of uncertainty. To guarantee r_C , a certain tolerable deviation is calculated by robustness function. While the same level of uncertainty provides the opportunity of securing returns up to r_W , however attaining those returns is not guaranteed. The decision maker can opt for a strategy to select

portfolio, depending upon its perception about the market and its nature. In case of uncertainty α , if a decision maker thinks that it could be pernicious, it would select a strategy to secure return r_C , whereas a propitious consideration of uncertainty would lead to strategy that may attain return as large as r_W .

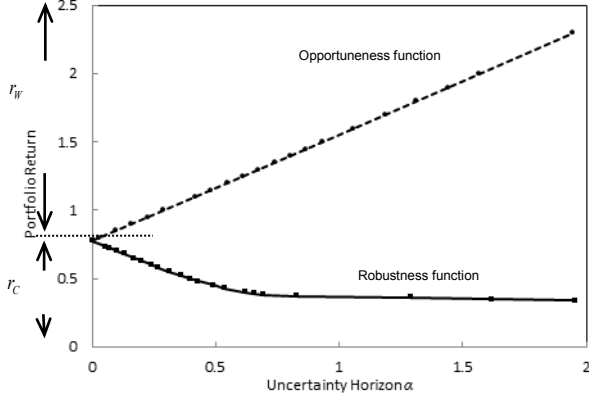


Fig. 3 Lower and upper bound of portfolio return for different

1) Robust Portfolio Selection

The critical return r_C is varied from 0.7785 to 0.34, for which the optimization results are shown in Figs. 3, 4 and 5. At $r_C = 0.7785$ the obtained robustness, *i.e.* allowable error is zero and varies from zero to 1.955 as r_C decreases from

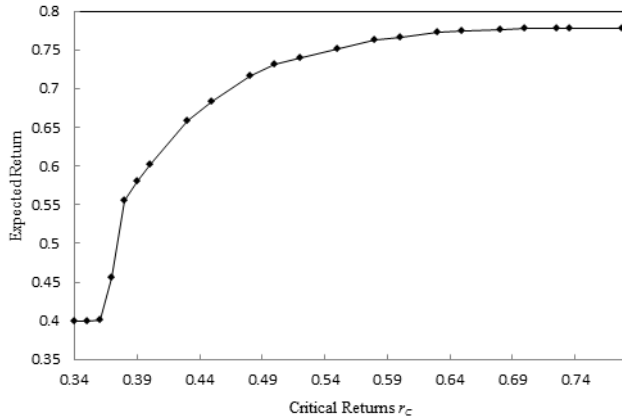


Fig. 4 Expected portfolio returns for different critical targeted returns

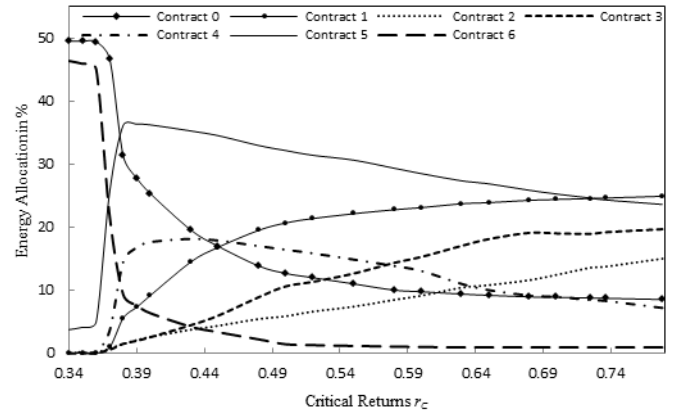


Fig. 5 Optimal energy allocation in different contracts for different critical returns

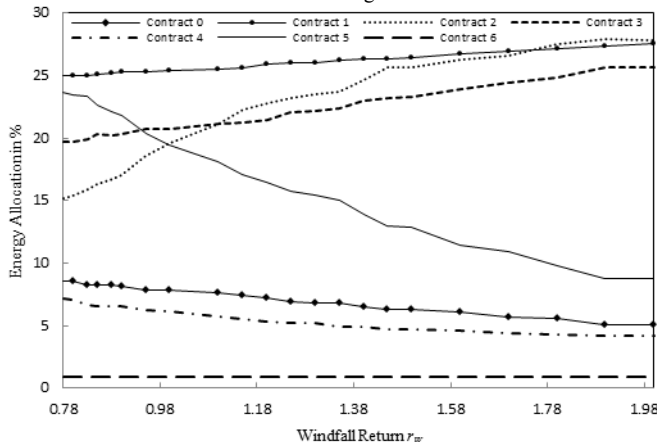


Fig. 6 Optimal energy allocation in different contracts for different windfall returns

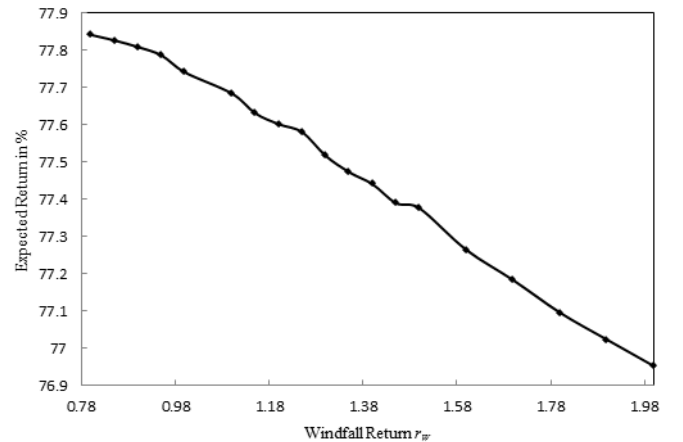


Fig. 7 Expected portfolio returns value for different windfall returns

0.7785 to 0.34, as shown in Fig. 3. Higher return targets are more demanding, thus robustness of the decision decreases with their increasing values. For each targeted return r_C , the robustness represents allowable uncertainty range α , under unfavorable deviation in market returns, up to which the decision can give return at least equal to r_C . For example, at $r_C = 0.6$ the obtained robustness $\alpha(w, 0.6) = 0.239$ represents that the allowable uncertainty/error for securing a minimum portfolio return of 0.6 is 0.239. The expected portfolio return $R(w(\alpha(r_C)), \tilde{r})$ also reduces with targeted returns, as shown in Fig. 4, but is always higher than the corresponding targeted value. For a certain value of r_C , the difference between expected portfolio return $R(w(\alpha(r_C)), \tilde{r})$ and maximum portfolio return $R_p(w, \tilde{r})$ is the cost which GenCo has to bear for robustness of selected decision.

With reducing return target r_C , the optimal energy allocation w varies in such a way that allocation increases in contracts with lesser variability and vice-versa. This happens because higher variability contracts are accompanied with higher possibility of losses and a strong risk-averse GenCo would aim to reduce its exposure towards losses. So, for a decision to be more robust, a GenCo allocates energy in Contracts 6 and 0, as observed from Fig. 5. Allocation in Contracts 4 and 5 increases initially due to their moderate

uncertain nature, but finally reduces for higher robustness. A trade-off exists between reward and robustness, *i.e.* a conservative GenCo aiming higher robustness has to compromise with returns.

2) Opportunistic Portfolio Selection

Opportunistic portfolio selection represents favorable face of uncertainty, reflecting the risk seeking behavior of an optimistic decision maker. This considers that uncertainty may provide opportunity for securing windfall returns. For windfall returns r_w varying from 0.7785 to 2.3, Figs. 3, 6 and 7 show the results of optimization for opportuneness $\beta(w, r_w)$ (35). $\beta(w, r_w)$ increases from zero to 1.955 with windfall returns as shown in Fig. 3. It means that opportunity of securing windfall benefits increases with uncertainty. To attain returns as large as r_w , GenCo has to tolerate a minimum uncertainty given by $\beta(w, r_w)$. Thus, if a GenCo desires higher windfall benefits, immunity of decision towards uncertainty reduces. For example, $\beta(w, r_w = 0.9) = 0.158$ means that a GenCo can attain return up to 0.90 from its trading portfolio, for the market returns rising up to 0.158 level. As the desire for windfall return increases, energy allocation increases in trades with higher variability and *vice-versa*, as shown in Fig. 6. This happens because contracts with higher variability have stronger possibility of favorable price spikes. Allocation in Contracts 1, 2 and 3 increases due to their higher variability, while allocation decreases in zero and low variability Contracts 0, 4 and 5. Allocation in Contract 6 remains nominal due to its low return and low variance.

An increasing desire for windfall returns creates a trade-off

between windfall and opportunity. It can be explained as follows: windfall returns are always accompanied with acceptance of higher uncertainty and this decreases the possibility of benefit from the opportunity arising due to favorable uncertainty. The expected returns $R(w(\beta(r_w)), \tilde{r}_j)$ are less than windfall returns, as shown in Fig. 7. If opportunistic strategy is selected and price spikes do not occur, then the portfolio return would be up to its expected value $R(w(\beta(r_w)), \tilde{r}_j)$. The difference between expected return and maximum return is the cost of enabling the possibility of higher desired benefit, which a GenCo would have to bear if market prices do not change favorably as per its aspiration.

The results highlight that opportunity of higher benefits increases with high variability contracts, conversely robustness of the decision increases with higher allocation in low variability contracts. Thus, there exists a trade-off between robustness and opportuneness; if any selected decision is highly robust it would not be opportunistic and a highly opportunistic decision would be least robust.

VI. FRAMEWORK VALIDATION

The proposed IGDT framework has been validated by assuming certain deviations in estimated returns. To analyze the robustness of risk averse decisions, market is assumed to go down, resulting in Δr decrement in returns. Δr is calculated using (27), for uncertainty levels α varying from zero to 1.955 (from robustness curve) and weights $w(\alpha(r_c))$ corresponding to critical returns r_c . These

TABLE IV
PORTFOLIO RETURNS USING ROBUSTNESS STRATEGY FOR DIFFERENT UNFAVORABLE PRICE MOVEMENTS BASED ON α

$\alpha \rightarrow$ $r_c \downarrow$	0.0000	0.1033	0.1972	0.2392	0.3137	0.3953	0.4307	0.6196	0.6946	0.8288	1.9550
0.78	0.7789	0.7195	0.6659	0.6420	0.5994	0.5529	0.5327	0.4249	0.3821	0.3056	-0.3370
0.70	0.7780	0.7200	0.6673	0.6437	0.6018	0.5560	0.5362	0.4300	0.3879	0.3126	-0.3200
0.63	0.7389	0.6826	0.6314	0.6085	0.5679	0.5235	0.5042	0.4012	0.3603	0.2873	-0.3265
0.60	0.7319	0.6772	0.6274	0.6052	0.5656	0.5224	0.5036	0.4035	0.3637	0.2926	-0.3043
0.55	0.7513	0.7001	0.6535	0.6327	0.5957	0.5552	0.5376	0.4439	0.4067	0.3401	-0.2188
0.50	0.7328	0.6851	0.6418	0.6225	0.5880	0.5504	0.5341	0.4469	0.4123	0.3504	-0.1694
0.48	0.6558	0.6200	0.5875	0.5730	0.5472	0.5190	0.5068	0.4414	0.4154	0.3690	-0.0207
0.40	0.6022	0.5736	0.5476	0.5360	0.5153	0.4928	0.4829	0.4306	0.4098	0.3727	0.0607
0.38	0.5545	0.5315	0.5105	0.5011	0.4844	0.4662	0.4583	0.4161	0.3994	0.3694	0.1178
0.37	0.4669	0.4542	0.4428	0.4376	0.4285	0.4186	0.4143	0.3912	0.3820	0.3656	0.2281
0.34	0.4005	0.3923	0.3847	0.3814	0.3754	0.3689	0.3660	0.3509	0.3549	0.3471	0.3438

TABLE V
PORTFOLIO RETURNS USING OPPORTUNITY STRATEGY FOR DIFFERENT FAVORABLE PRICE MOVEMENTS BASED ON β

$\alpha \rightarrow$ $r_w \downarrow$	0.0000	0.0936	0.2884	0.4820	0.6104	0.6747	0.9305	1.1855	1.4396	1.5669
0.78	0.7789	0.8318	0.9430	1.0535	1.1267	1.1634	1.3094	1.4549	1.5999	1.6725
0.85	0.7787	0.8321	0.9442	1.0555	1.1294	1.1663	1.3135	1.4601	1.6063	1.6795
1.00	0.7779	0.8322	0.9460	1.0591	1.1342	1.1717	1.3213	1.4703	1.6188	1.6931
1.15	0.7768	0.8317	0.9469	1.0614	1.1374	1.1753	1.3267	1.4775	1.6277	1.7030
1.25	0.7760	0.8313	0.9475	1.0630	1.1395	1.1778	1.3304	1.4825	1.6340	1.7099
1.30	0.7758	0.8320	0.9437	1.0547	1.1283	1.1652	1.3119	1.4580	1.6037	1.6767
1.50	0.7741	0.8303	0.9480	1.0650	1.1427	1.1815	1.3361	1.4902	1.6438	1.7208
1.70	0.7724	0.8290	0.9479	1.0660	1.1443	1.1835	1.3396	1.4951	1.6502	1.7278
1.90	0.7705	0.8276	0.9476	1.0669	1.1460	1.1856	1.3432	1.5002	1.6567	1.7351
2.00	0.7702	0.8274	0.9475	1.0668	1.1460	1.1856	1.3433	1.5004	1.6571	1.7355

calculated values of Δr are used in (18) to evaluate actual returns r , to finally obtain portfolio return $R_p(w, r)$ using (17), and shown in Table IV. Each row represents a strategy corresponding to return target r_c , while each column represents an uncertainty level. Thus, diagonal elements are portfolio returns for $\alpha = \alpha(w, r_c)$, with maximum sustainable uncertainty for trading strategy corresponding to r_c . The lower triangular region represents robust portfolio returns. Each row of Table IV represents that if market returns fall within the range defined by robustness region, portfolio return would never be less than critical return r_c . Each column of Table IV represents that for a certain fall in market prices, portfolio return would always be higher than critical return, for all those strategies which cover that fall defined by robustness region.

To analyze opportuneness in risk seeking decision, it is assumed that market prices would favorably increase and returns would rise up to \tilde{r} . The value of Δr is calculated for different α (zero to 1.955 from opportuneness curve) and trading strategy $w(\beta(r_w))$, corresponding to desired windfall returns r_w . These are used to calculate portfolio return $R_p(w, r)$ from (17), and are shown in Table V. Diagonal elements of the table represent portfolio returns obtained with $\alpha = \beta(w, r_w)$ and trading strategy corresponding to r_w . Thus, by selecting opportunity strategy, GenCo can have returns as large as r_w , if market moves favorably up to $\beta(w, r_w)$. The selected strategy can enhance portfolio returns, with increasing market movement.

VII. CONCLUSION

This paper considers GenCo's trading portfolio optimization for pool and bilateral markets, involving congestion and pool price uncertainties. Conventional decision making approaches are based on estimated market returns, which may vary from actual ones, and are unable to tackle severe uncertainty. To deal with this uncertainty, an Info-gap decision theory framework has been developed by quantifying information gap existing between estimated and actual values. Compared to conventional portfolio theory, this formulation offers decisions that are robust towards losses and opportunistic towards capturing higher gains.

The proposed framework has been validated by assuming deviations in return estimations. The results from practical case study illustrate that the proposed approach can guarantee portfolio return under unfavorable price change within the robustness region. Also, it enables a GenCo to take advantage of opportunity for attaining windfall returns, caused due to favorable price spikes. The results provide a range of decisions for GenCo to select the most appropriate. These decisions are evaluated for different criteria, such as trade-offs

existing between reward and robustness, opportunity and windfall gain, and robustness and opportuneness, for optimal portfolio selection. To secure a minimum level of benefits, as well as to capture higher gains, a GenCo has to bear some cost, depending on its desire for robustness or opportunity characteristics of the selected decision. This work can be extended for trading decision making of GenCo in different markets influenced by external market uncertainties, and involve multiple types of contracts.

VIII. ACKNOWLEDGEMENTS

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IX. APPENDIX A

$$\min_{\Delta r} \{w^T \Delta r : \Delta r^T C^{-1} \Delta r \leq \alpha^2\}$$

As the minimization problem is convex, Lagrange method is applied for optimization. So, the first order optimality condition for the associated Lagrangian problem is

$$\begin{aligned} \nabla_{\mu, \Delta r} \{w^T \Delta r + \mu \{\alpha^2 - \Delta r^T C^{-1} \Delta r\}\} &= 0 \\ (w^T - 2\mu C^{-1} \Delta r^T, \quad \alpha^2 - \Delta r^T C^{-1} \Delta r) &= (0, 0) \end{aligned}$$

Hence,

$$\Delta r^T = \frac{w^T C}{2\mu} \text{ and } \alpha^2 = \Delta r^T C^{-1} \Delta r$$

After substituting value of Δr

$$\begin{aligned} \alpha^2 &= \frac{w^T C}{2\mu} C^{-1} \frac{wC}{2\mu} = \frac{1}{4\mu^2} w^T C w \\ \Rightarrow \frac{1}{2\mu} &= \pm \frac{\alpha}{\sqrt{w^T C w}} \\ \Rightarrow \Delta r &= \pm \alpha \frac{wC}{\sqrt{w^T C w}} \end{aligned}$$

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XI. BIOGRAPHIES



Parul Mathuria received the B.E. in Electrical Engg. from University of Rajasthan, and M.Tech. in Power System Engg. from Malaviya National Institute of Technology, all in Jaipur, India. She is presently pursuing Ph.D. from Malaviya National Institute of Technology Jaipur. Her main area of interest is Power Markets and Risk Management.



Rohit Bhakar (M'10) received the B.E. in Electrical Engg. from M.B.M. Engg. College Jodhpur, M.Tech. in Power System Engg. from Malaviya National Institute of Technology Jaipur, and Ph.D. from Indian Institute of Technology Roorkee, all in India. He has worked as an Assistant Professor with Malaviya National Institute of Technology Jaipur. Presently, he is a Prize Fellow in Energy Demand Reduction at Department of Electronic and Electrical Engineering, University of Bath, UK. His research interests are in Power System Economics, Network Pricing, Electricity Markets and Energy Storage.